Random Graphs
Exercise Sheet 5

Question 1. Show that the hitting time for the existence of a matching in a bipartite graph is the same as the hitting time for having minimum degree at least one.

Question 2. Let r be fixed. Determine a threshold (ideally as in Theorem 6.9) for having a collection of r edge-disjoint perfect matchings in $G_{n,p}$.

(*What about a collection of r edge-disjoint Hamiltonian cycles?)

Question 3. Prove Claim 19 and Claim 20 from the proof of Theorem 6.13

Question 4. Show that Theorem 6.13 also holds for the bipartite random graph model $G_{n,n,p}$

Question 5. Show that if 3 divides n and $p = \omega\left(\sqrt{\frac{\log n}{n}}\right)$ then with high probability $G_{n,p}$ can be covered by $\frac{n}{3}$ disjoint triangles.

Question 6. Consider a random digraph $D_{n,p}$ with vertex set $[n]$ where each directed edge (x, y) is included independently with probability p. Show that, for every n and p

$$\mathbb{P}(D_{n,p} \text{ has a directed Hamiltonian cycle}) \geq \mathbb{P}(G_{n,p} \text{ has a Hamiltonian cycle})$$